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DESIGN OF BIASING MAGNETS FOR E-F AND I-J BAND CROSS-FIELD AMPL--ETC(U)  
JUN 78 H A LEUPOLD, F ROTHWARTH

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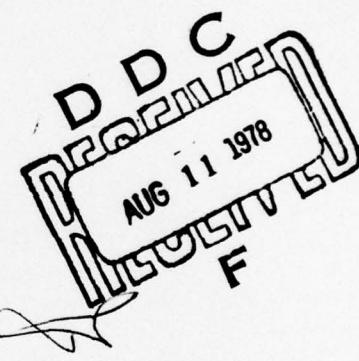
DESIGN OF BIASING MAGNETS FOR E-F AND I-J BAND  
CROSS-FIELD AMPLIFIER TUBES

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H. A. Leupold  
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Electronics Technology & Devices Laboratory

June 1978



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## CONTENTS

	<u>Page</u>
INTRODUCTION	1
PERMEANCE CALCULATIONS	1
CALCULATION OF THE GAP FIELDS	9
Configuration A	9
Configuration B	10
Configuration C	12
CONCLUSIONS	14
APPENDIX A	16
APPENDIX B	18

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## FIGURES

1. Maximum allowable dimensions for the E-F band cross-field amplifier.	2
2. Conventionalized flux paths used to approximate the actual magnetic field configuration for the purpose of calculating the relative permeances for the alternative magnetic circuits A and B.	4
3. Configuration C.	13
A-1. Determination of angle $\theta$ .	16

## TABLES

1. Magnetic Circuit Permeances for Configurations A, B and C.	8
2. Magnetomotive Forces (F) and Gap Fields ( $H_g$ ) for Various Magnets in Configurations A, B and C.	11

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## DESIGN OF BIASING MAGNETS FOR E-F AND I-J BAND CROSS-FIELD AMPLIFIER TUBES

### INTRODUCTION

A compact, lightweight permanent magnet circuit for an E-F band cross-field amplifier is needed for certain airborne devices. The space limitations are as shown in Fig. 1 and the field required in the gap is from 2000 to 2500 oersteds. The severe space and weight limitations, as well as the fairly high field needed in the relatively large gap, lead one to believe that rare earth-cobalt magnet materials might be the only ones that can satisfy these constraints. Their high energy products, high coercivities, reversible demagnetization characteristics and the possibility of designing them to have thermal stability make their use desirable.<sup>1</sup> The present design calculation is done to determine the energy product that would be required of a rare earth-cobalt material to satisfy the above gap and field requirements.

### PERMEANCE CALCULATIONS

The design calculations are straightforward and follow the method of "estimation of permeances" which is described in detail elsewhere.<sup>2-7</sup> This method makes use of the analogy between electric and magnetic circuits where the magnetic Ohm's law reads

$$\phi = PF \quad (1)$$

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1. F. Rothwarf, H.A. Leupold, and L.J. Jasper, Jr., "Millimeter-Wave Micro-wave Device Applications of Rare Earth-Cobalt Magnets," Proceedings of the Third International Workshop on Rare Earth-Cobalt Magnets and Their Applications, Paper No. V-1, University of California, San Diego, 1978.
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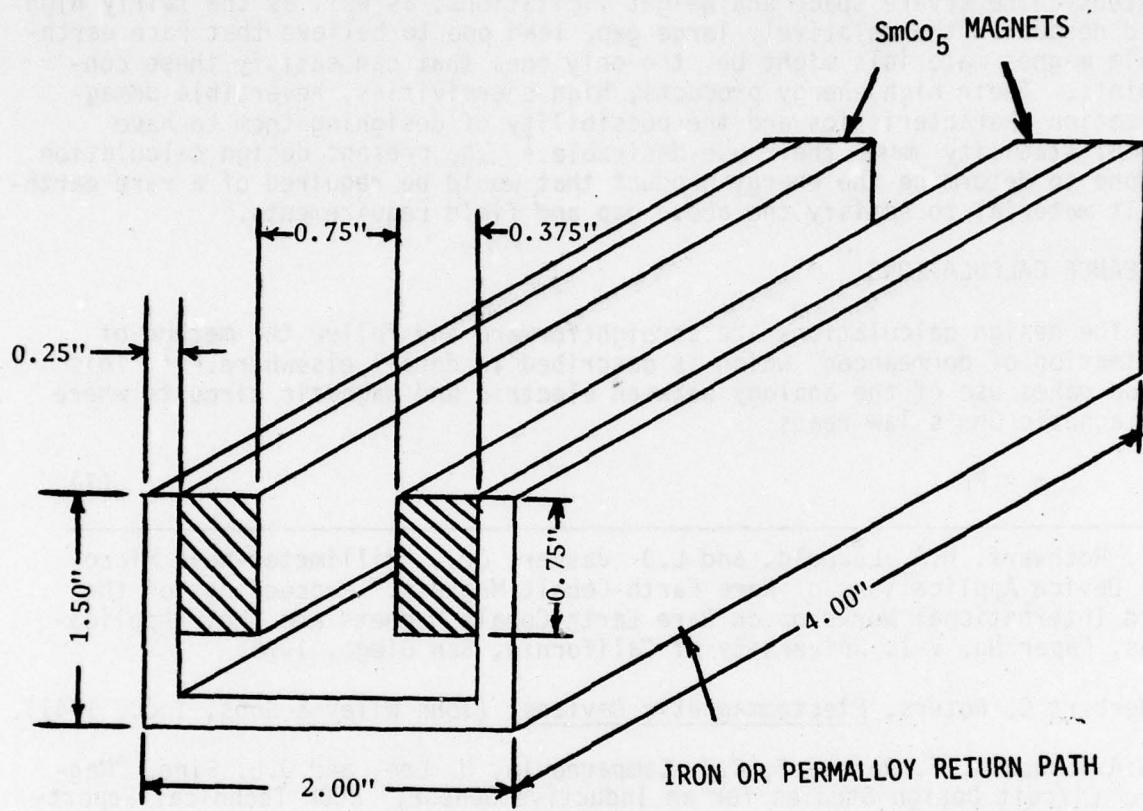


Fig. 1. Maximum allowable dimensions for the E-F band cross-field amplifier. The width and height restrictions are removed for the I-J band device but the overall length is limited to three inches.

and the flux  $\phi$ , the permeance  $P$  and the magnetomotive force  $F$  are the magnetic analogues of the respective electrical quantities, current  $I$ , conductance  $G$  and electromotive force  $V$ .

The magnetomotive force (MMF) of a magnet is given by

$$F = \frac{L_M B_r}{\mu_R} \quad (2)$$

where  $L_M$  is the length of the magnet,  $B_r$  its remanence, and  $\mu_R$  the reversible permeability which, in the case of a good  $\text{SmCo}_5$  magnet, is simply the slope of its linear demagnetization curve, typically between 1.00 and 1.05. As in an electrical circuit, the permeance of a magnetic circuit is composed of a load permeance and a source permeance. The latter is just the internal permeance of the magnet given by

$$P_M = \frac{A_M \mu_R}{L_M} \quad (3)$$

where  $A_M$  is the area of the magnet's cross section.

The external permeance paths are approximated by dividing the space around the magnet into regions bounded by cylindrical and spherical surfaces connecting points of different magnetic potential. The procedure is illustrated in Fig. 2 for two alternative circuit designs (configurations A and B). The first design Fig. 2A is the same as shown in Fig. 1 where the two  $\text{SmCo}_5$  magnets constitute the poles of the 0.75-inch gap with flux being carried by an iron or permalloy yoke. The second design is shown in Fig. 2B where a single  $\text{SmCo}_5$  magnet is fitted with two soft iron poles that define a gap with the same dimensions as in Fig. 1. Points of demarcation between flux paths are found by a method illustrated with reference to Fig. 2A. In this configuration one must find the location of point A which is the demarcation between flux paths  $P_{MS}^3$  and  $P_{MK}$ . This is done by comparing the length of arc AB with that of straight line segment AC. A is then located so that  $AB=AC$ . If B and C were at different magnetic potentials  $V_B$  and  $V_C$ , then the condition for locating A would become

$$\frac{V_B}{AB} = \frac{V_C}{AC}$$

$$\bar{H}_{AB} = \bar{H}_{AC} \quad (4)$$

where  $\bar{H}_{AB}$  and  $\bar{H}_{AC}$  are the average fields along the flux lines AB and AC respectively.

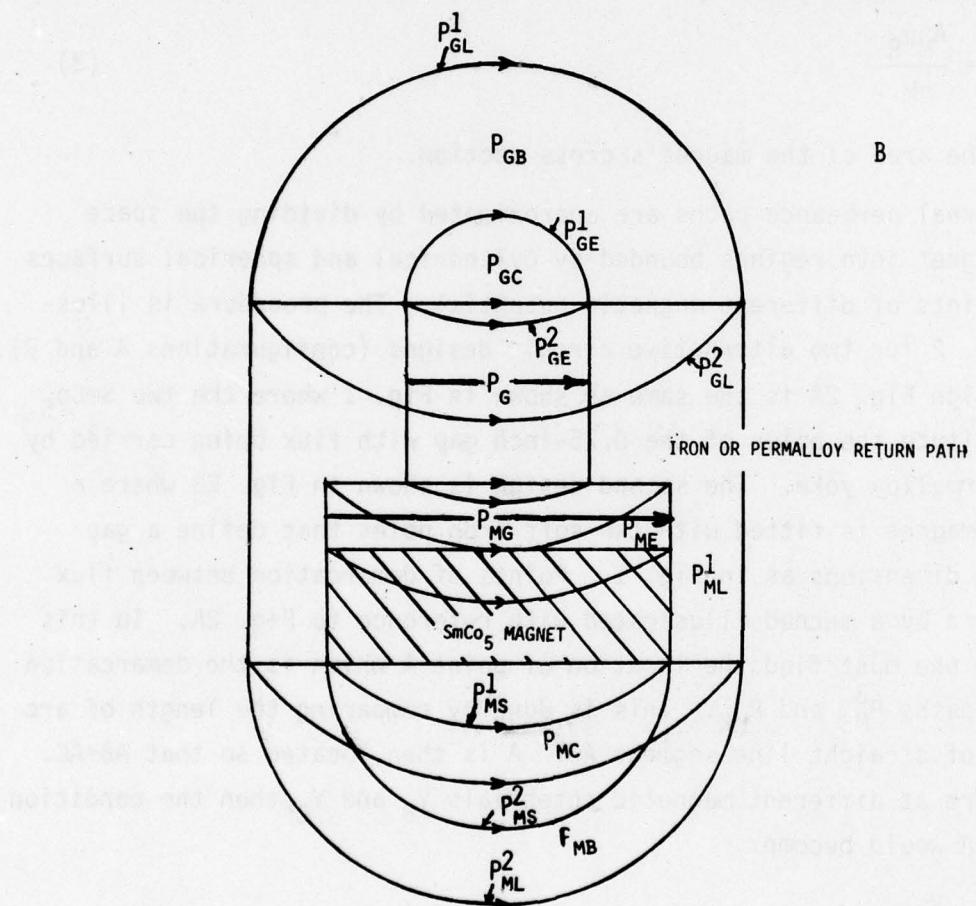
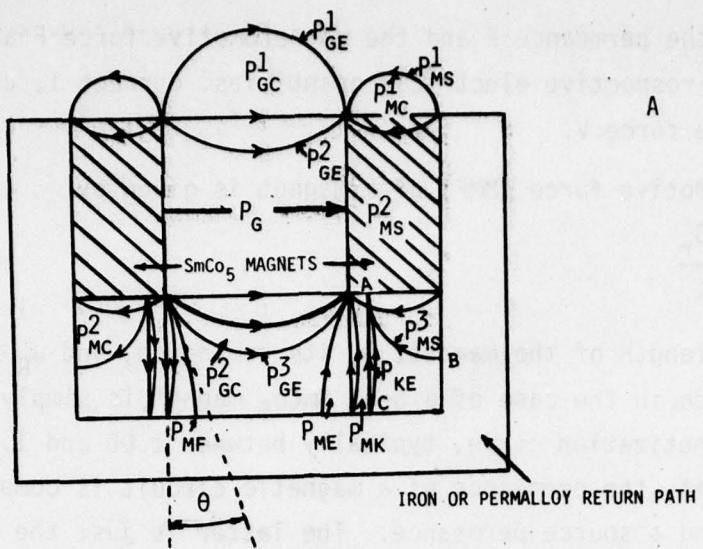


FIG. 2. CONVENTIONALIZED FLUX PATHS USED TO APPROXIMATE THE ACTUAL MAGNETIC FIELD CONFIGURATION FOR THE PURPOSE OF CALCULATING THE RELATIVE PERMEANCES FOR THE ALTERNATIVE MAGNETIC CIRCUITS A AND B.

The permeances of most of the paths illustrated in Fig. 2 are standard forms for which there are published formulae. The nonstandard permeances can be estimated either through modification of the formulae for similar standard paths or from first principles. The following pertinent formulae are given with reference to Fig. 2.

A. Permeances of gaps with opposing parallel planar faces:  $P_{MK}$  of Fig. 2A,  $P_{MG}$  of Fig. 2B, and  $P_G$  of both configurations are such permeances and are given by the expression

$$P = \frac{A_g}{l_g g}$$

where  $l_g$  is the gap length and  $A_g$  its cross-sectional area.

B. Permeances between parallel, linear edges of opposing surfaces such as  $P_{GE}^1$ ,  $P_{GE}^2$ , and  $P_{KE}$  are given by

$$P = 0.26 L$$

where  $L$  is the length of the gap edge.  $P_{GE}^3$  is only partially effective because of partial preemption by path  $P_{ME}$ . By use of Eq. (4) we find that the angle  $\theta$  is  $16.0^\circ$  rather than  $0^\circ$  as would be the case if  $P_{GE}^3$  were fully operative (see Appendix A). We approximate  $P_{GE}^3$  by a plausible modification of the standard form appropriate when  $\theta=0$ .

$$\frac{90^\circ - 16^\circ}{90^\circ} 0.26 L = (0.82) 0.26 L$$

The rationale for this procedure is that the average cross section of path  $P_{GE}^3$  is roughly proportional to the angle of departure of the path boundary. Ordinarily this angle would be  $90^\circ$  with respect to the lower edges of the magnets. But, because of partial preemption by path  $P_{ME}$ , it is only  $(90^\circ - 16^\circ)$  in this case. Since the average flux line length does not vary much with angle, the permeance would be directly proportional to the cross-section area and hence to the angle of departure.

C. Permeances of annular, cylindrical sections extending between coplanar surfaces across a gap. There are no such flux paths in configuration A, but they are represented in configuration B by  $P_{GL}^1$ ,  $P_{GL}^2$ ,  $P_{ML}^1$ ,  $P_{ML}^2$ . The formula is

$$P = \frac{L}{\pi} \ln \left( 1 + \frac{2t}{g} \right)$$

where  $L$  is the length of the annular section,  $t$  its thickness, and  $g$  its inner diameter.

D. Permeances of spherical quadrants ("orange slices") connecting opposing corners of opposing parallel surfaces and given by  $P=0.077 L$ .  $P_{GC}^1$  of A and  $P_{GC}$  of B are such permeances.  $P_{GC}^2$  of A is only partially effective due to interference from paths  $P_{ME}$  and  $P_{MF}$ . In B,  $P_{GC}^2$  is inoperative, being preempted by  $P_{ME}$ . In section B it was shown that to find  $P_{GE}^3$  one had to multiply the standard formula by a factor 0.82 to account for partial preemption. In the case of  $P_{GC}^2$ , however, the preemption affects two dimensions so the appropriate factor must be  $(0.82)(0.82) = 0.68$  and

$$P_{GC}^2 = 2(0.68)(0.077) L$$

where the factor of 2 is included to account for the two such paths present.

E. Permeances of paths that are quadrants of spherical shells ("melon slices") connecting colinear edges separated by a gap.  $P_{GB}$  and  $P_{MB}$  of configuration B are such paths and the formula is

$$P = \frac{t}{4}$$

where  $t$  is the thickness of the shell.

F. Permeances of cylindrical quadrants connecting edges with plane surfaces parallel to these edges.  $P_{ME}$  is such a path which is only  $16^\circ/90^\circ = 0.178$  effective due to preemption by other paths. Multiplying this factor by the standard form

$$P = 0.52 L \text{ yields}$$

$$P = (0.178)(0.52)L$$

G. Permeance of  $P_{MF}$ . This is a nonstandard path enclosed by flux lines extending from the corner of origin of  $P_{ME}$  of Fig. 2A to the edge of the keeper trough. An average of the formulae  $0.0770 L$  and  $0.0385 L$  seems reasonable for the calculation of  $P_{MF}$ . The expression must be further modified by the factor  $\left(\frac{16^\circ}{90^\circ}\right)^2$  because the angle of departure from the corner is  $16^\circ$  rather than  $90^\circ$  with the square occurring because two dimensions are affected. We finally have

$$P_{MF} = (0.062)(0.058) L = 0.0036 L$$

H. Semi- or quarter-cylindrical flux paths backstreaming from sides of magnets.  $P_{MS}^1$ ,  $P_{MS}^2$  and  $P_{MS}^3$  are such paths. The permeances are

$$P = 0.318 y$$

where  $y$  is the width of the magnet face.

I. "Orange slice" backstreaming paths from longitudinal edges of magnets  $P_{MC}^1$ ,  $P_{MC}^2$  and  $P_{MC}^3$  are such paths and the permeances are given by

$$P = \frac{L_M}{8}$$

where  $L_M$  is the length of the magnet.  $P_{MC}^2$  is only of this standard form in the plane perpendicular to the figure. As one rotates the plane of the flux lines to the plane of the paper and coincidence with the boundary of  $P_{MS}^3$ , the flux lines go from semicircular to quarter-circular form. So an average value between these two types of paths seems reasonable and we have

$$P_{MC}^2 = \frac{\frac{L}{8} + \frac{L}{4}}{2} = 0.188 L$$

where  $L$  is the radius of the arc bounding  $P_{MS}^3$ .

J. The "condenser" permeance  $P_{MK}$  is essentially of the same form as permeances  $P_G$  except that since one side of the gap is the wall of a magnet, account must be taken of the fact that the path is not driven by the entire MMF of the magnet. This is done by multiplying by a factor (see Appendix B)

$$f = \frac{2L_M - L_s}{4L_M}$$

where  $L_M$  is the length of the magnet and  $L_s$  the width of the path  $P_{MK}$

$$P_{MK} = f \frac{L_s L_w}{L}$$

where  $L_w$  is the length of the path perpendicular to the plane of Fig. 2A and  $L$  is the gap length of the path.

The values of these permeances for configurations A and B are listed in Table 1.

Where there are several parallel paths of the same type, the permeance listed for that type is the sum of the equal contributions of the individual paths.

TABLE 1  
Magnetic Circuit Permeances for Configurations A, B and C

	<u>A</u>	<u>B</u>	<u>C</u>
$P_G$	4.00	4.00	4.50
$P_{MK}$	0.414	--	0.891
$P_{MG}$	--	0.718	--
$P_{GE}^1$	1.04	1.04	0.780
$P_{GE}^2$	0.390	0.390	0.390
$P_{GE}^3$	0.855	--	0.780
$P_{GL}^1$	--	1.25	0.339
$P_{GL}^2$	--	0.468	0.170
$P_{ML}^1$	--	0.314	--
$P_{ML}^2$	--	0.0785	--
$P_{GC}^1$	0.115	0.115	0.0770
$P_{GC}^2$	0.079	--	0.0770
$P_{GB}$	--	0.313	0.0540
$P_{MB}$	--	0.110	--
$P_{ME}$	0.370	--	--
$P_{MF}$	0.004	--	--
$P_{MS}^1$	1.27	1.27	0.716
$P_{MS}^2$	0.477	0.299	0.358
$P_{MS}^3$	2.19	--	1.217
$P_{MC}$	--	0.195	--
$P_{MC}^1$	0.094	--	0.094
$P_{MC}^2$	0.120	--	0.260
$P_{KE}$	0.027	--	0.0772
$P_t^E$	8.96	10.7	9.32
$P_t^M$	4.20	1.27	2.36

## CALCULATION OF THE GAP FIELDS

### Configuration A

From Fig. 2 and Table 1 we can obtain the total load permeance  $P_t^E$  by the usual rules of combination of parallel and series permeances.

From Table 1 we see that the total load reluctance obtained in this manner for configuration A is

$$R_t^E = \frac{1}{P_t^E} = \frac{1}{8.96} = 0.112 \text{ inch}^{-1}$$

The internal reluctance of the magnets is just the sum of those of the individual magnets since they are in series. Thus,  $R_t^M = 2/P_M^1 = 2L_M/A_M\mu_R$

and  $R_t^M = 2 \frac{0.375}{(1.05)(3)} = 0.238 \text{ inch}^{-1}$

and the total circuit reluctance is

$$R_t = R_t^M + R_t^E = 0.238 + 0.112 = 0.350 \text{ inch}^{-1}$$

The MMF of the combined magnets is also the sum of the MMF's of the individual magnets

$$F = 2 \frac{L_m B_r}{\mu_R} = 2 \frac{(0.375)(2.54)}{1.05} B_r = 1.81 B_r, \text{ kilogilberts}$$

where the factor 2.54 converts inches to centimeters. The total flux is then given by the magnetic Ohm's law, (again the conversion from inches to cm is used

$$\phi_t = \frac{F}{R_t} = \frac{(2.54)(1.81)}{(0.350)} B_r = 13.1 B_r, \text{ kilomaxwells}$$

Since all of the circuit permeances except for the source permeance are in parallel, the fraction of  $\phi_t$  passing through the gap is given by

$$\frac{P_G}{P_t^E} = \frac{4.00}{8.96} = 0.446$$

and  $\phi_G = 0.446 \phi_t = (0.446)(13.1) B_r = 5.84 B_r, \text{ kilomaxwells}$

The 4 inch x 3/4 inch cross-section area of the gap space is

$$A_G = (4)(0.75)(2.54)(2.54) = 19.35 \text{ cm}^2$$

and the flux density in the gap is

$$B_G = \frac{\phi_G}{A_G} = \frac{5.84 B_r}{19.35} = 0.302 B_r, \text{ kilogauss (kG)}$$

If  $B_r$  is 9.00 kG, as in the best of commercially available  $\text{SmCo}_5$  magnets, the useful flux density would be

$$B_G = (0.302)(9.00) = 2.72 \text{ kG}$$

which exceeds the desired range of 2.0 to 2.5 kG (see Table 2). Typical commercial  $\text{SmCo}_5$  magnets have  $B_r$ 's of around 7.00 kG with some falling as low as 5.5 kG; well below the minimum of 6.7 kG required for a gap field of 2.0 kG. For this reason the use of  $\text{SmCo}_5$  as the active magnet material in the present design must be considered marginal.

### Configuration B

Because of the rather large ratio of gap length to magnet length in configuration A, it was feared that perhaps there would be excessive backstreaming from the magnet walls as well as inordinate magnet to yoke leakage via permeance paths  $P_{EK}$ ,  $P_{MF}$ ,  $P_{MK}^3$ ,  $P_{MS}^3$  and  $P_{MC}^2$ . For this reason, gap field calculations were made for configuration B to check for the possibility of improvement over A.

Here the total external reluctance is  $0.0934 \text{ inch}^{-1}$  and the source reluctance

$$R_M = \frac{L_M}{A_M \mu_R} = \frac{1.56}{(0.47)(4)(1.05)} = 0.790 \text{ inch}^{-1}$$

The total circuit reluctance is then

$$R_M + R_t^E = 0.0934 + 0.790 = 0.883 \text{ inch}^{-1} = 0.348 \text{ cm}^{-1}$$

the MMF

$$F = \frac{(1.56)(2.54)}{1.05} B_r = 3.77 B_r, \text{ kilogilberts}$$

the flux

$$\phi_t = \frac{3.77 B_r}{0.348} = 10.8 B_r, \text{ kilomaxwells}$$

the gap flux

$$\phi_G = \frac{P_G}{P_t^E} \phi_t = \frac{(4.00)(10.8)}{10.7} B_r = 4.04 B_r, \text{ kilomaxwells}$$

TABLE 2  
Magnetomotive Forces (F) and Gap Fields ( $H_g$ ) for Various Magnets in  
Configurations A, B and C

Magnet Material	$B_r$	A			B			C		
		F	$H_g$	F	$H_g$	F	$H_g$	F	$H_g$	
Best Commercial SmCo <sub>5</sub>	9.0 kG	16.3	2.7	45.3	1.9	21.8	(43.5)	3.5	(4.8)	
Typical Commercial SmCo <sub>5</sub>	7.0 kG	12.7	2.1	35.2	1.5	16.9	(33.8)	2.7	(3.7)	
Best Laboratory SmCo <sub>5</sub>	10.7 kG	19.4	3.2	53.9	2.2	25.9	(51.7)	4.1	(5.7)	
Best Laboratory Sm <sub>2</sub> (Co,TM) <sub>17</sub>	11.2 kG	20.3	3.4	56.4	2.3	27.2	(54.2)	4.3	(5.9)	

F's are in kilogausses.

$H_g$ 's are in kOe.

The value of  $\mu_R = 1.05$  is assumed for all magnets.

The numbers in parentheses refer to the modified configuration C in which the magnets are twice as long and twice as high as in the original configuration.

and the gap flux density

$$B_G = \frac{\phi_G}{A_G} = \frac{4.04 B_r}{19.4}$$

$$B_G = 0.208 B_r, \text{ kG}$$

Again assuming  $B_r = 9.0 \text{ kG}$

$$B_G = 1.87 \text{ kG}$$

which is not as efficient as configuration A and falls short of the required 2.0 kG.

### Configuration C

For the I-J band amplifier the field requirements are 5 kG in a 0.5-inch gap to be obtained from a package of the same external dimensions as configuration A except that the overall length is to be reduced to 3 inches.

The permeances are of the same form as in configuration A except that now the arc forming the boundary of  $P_{GE}^1$  would no longer be twice that bounding  $P_{MS}^1$  so that the condition  $F(P_{GE}^1) = F(P_{MS}^1)$  no longer holds. This means that the demarcation between the gap spanning permeances and those encompassing the backstreaming flux lines would no longer be the peripheral edge of the magnet face. The new demarcation must be at B on the magnet wall as shown in Fig. 3 which also illustrates the new permeances  $P_{GL}^2$ ,  $P_{GL}^1$  and  $P_{GB}$  arising in configuration C.

The location is found through the competition condition

$$\frac{F_{AB}}{AB} = \frac{F_{BC}}{BC}$$

Because arc AB is driven by both magnets and arc BC is driven by only one magnet  $F_{AB} = 2F_{BC}$ , thus

$$\frac{2F_{BC}}{AB} = \frac{F_{BC}}{BC}$$

$$2BC = AB$$

$$\pi (L_M - x) = \pi \left( \frac{\ell_g}{2} + x \right)$$

where x is as shown in Fig. 3.

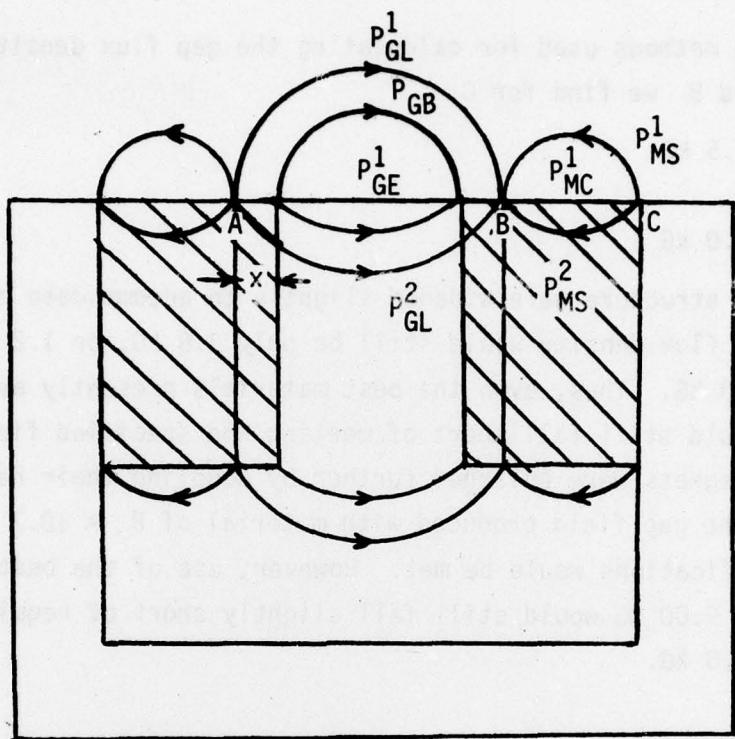


Fig. 3. Configuration C. This arrangement differs from configuration A chiefly in its shorter gap which gives rise to the additional permeance paths  $P_{GL}^1$ ,  $P_{GL}^2$ ,  $P_{GB}$ . The location of points A and B and length x are derived in the text.

$$L_M - \frac{g}{2} = 2x$$

$$x = \frac{L_M}{2} - \frac{g}{4}$$

and similar adjustments must be made for permeances at the corners where a "melon slice" permeance  $P_{GB}$  appears as well as for the dimensions of permeance paths  $P_{KE}$ ,  $P_{MK}$ , etc.

By the same methods used for calculating the gap flux densities of configurations A and B we find for C

$$B_G = 3.5 \text{ kG}$$

if

$$B_r = 9.0 \text{ kG}$$

Even if the structure were widened slightly to accommodate one-inch magnets, the gap flux density would still be only 3.8 kG, or 1.2 dG less than the required 5.00 kG. Thus, even the best materials presently available with  $B_r \approx 10.7 \text{ kG}$ , would still fall short of meeting the specified field requirements. If the magnets were enlarged further by doubling their height from 0.75" to 1.5", the gap field produced with material of  $B_r = 10.7 \text{ kG}$  would be 5.7 kG and specifications would be met. However, use of the best commercial material of  $B_r = 9.00 \text{ kG}$  would still fall slightly short of requirements with a gap field of 4.8 kG.

#### CONCLUSIONS

We have seen that presently available  $\text{SmCo}_5$  permanent magnets can fulfill the flux density requirements of the proposed E-F band amplifier without exceeding space limitations if care is taken to use only material with  $B_r > 7.00 \text{ kG}$ . For I-J band applications, the structure would have to be more than doubled in cross-sectional area and even then the best of commercial  $\text{SmCo}_5$  would have to be used to barely meet the specified flux-density of 5000 gauss.

Improved quality control and upgrading of the commercial  $\text{SmCo}_5$  magnets to achieve laboratory results could conceivably be a marginally satisfactory solution. A better approach would be to employ magnetic materials with higher moments such as the  $\text{Sm}_2(\text{Co},\text{TM})_{17}$  compounds, should it prove practicable to mass-produce such permanent magnets of sufficient intrinsic coercivity, with linear, reversible demagnetization curves and remanences greater than 11 kG.

The Japanese recently synthesized promising material which showed a remanence of 11.2 kG and linear reversibility to a field  $\sim 5$  kOe.<sup>8</sup> Table 2 shows that a magnet composed of such material meets the specifications for the I-J band amplifier (configuration C improved). Again the performance would be only marginal with little leeway to allow for even slightly substandard  $B_r$ 's. It would also be desirable for the intrinsic coercivity to be sufficiently high to insure linearity of the demagnetization curve to negative fields of at least 11.2 kOe. This would insure virtual immunity to demagnetization by exposure to stray fields or through use in devices requiring very unfavorable magnet aspect ratios. To make the I-J band circuit truly viable one should employ a magnet of  $B_r$  of at least 12 kG, a  $\mu_R$  as close to 1.0 as possible, and an  $I_{cH}$  of at least 15 kOe. These calculations point to the growing need for more R&D in the United States to develop such high energy product, high coercivity magnet materials. A more comprehensive review of the millimeter-wave and microwave requirements for such materials is to be found in a recently completed paper by Rothwarf, Leupold and Jasper.<sup>9</sup>

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8. T. Ojima, S. Tomizawa, T. Yoneyama and T. Hori, "New Type Rare Earth Cobalt Magnets with an Energy Product of 30 MGoe," Japan J. Appl. Phys. 16, 671 (1977).

9. See reference 1, page 1.

## APPENDIX A

### THE DETERMINATION OF ANGLE $\theta$ FOR FLUX PATH $P_{GE}^3$

Equation (4) shows that the condition determining the angle  $\theta$  in Fig. 2A is that the length of arc  $\widehat{CD}$  be equal to twice arc  $\widehat{CE}$ . The factor of two arises because arc  $\widehat{CD}$  is driven by the full MMF of the magnetic circuit (2 magnets) while arc  $\widehat{CE}$  is driven by a single magnet only. We must now express the lengths of both arcs in terms of  $\theta$  and insert them in the expression

$$\widehat{CD} = 2\widehat{CE} \quad (1A)$$

We proceed with reference to Fig. A-1 which is constructed so that arcs  $\widehat{CD}$  and  $\widehat{CE}$  are tangent with  $\widehat{CE}$  normal to the top base of the permalloy yoke. In this way we satisfy our original conditions of circular flux lines at the boundaries of permeance paths, normal intersections of flux lines with surfaces and uninterrupted emanation of flux lines from the surfaces, corners, and edges of the magnets.

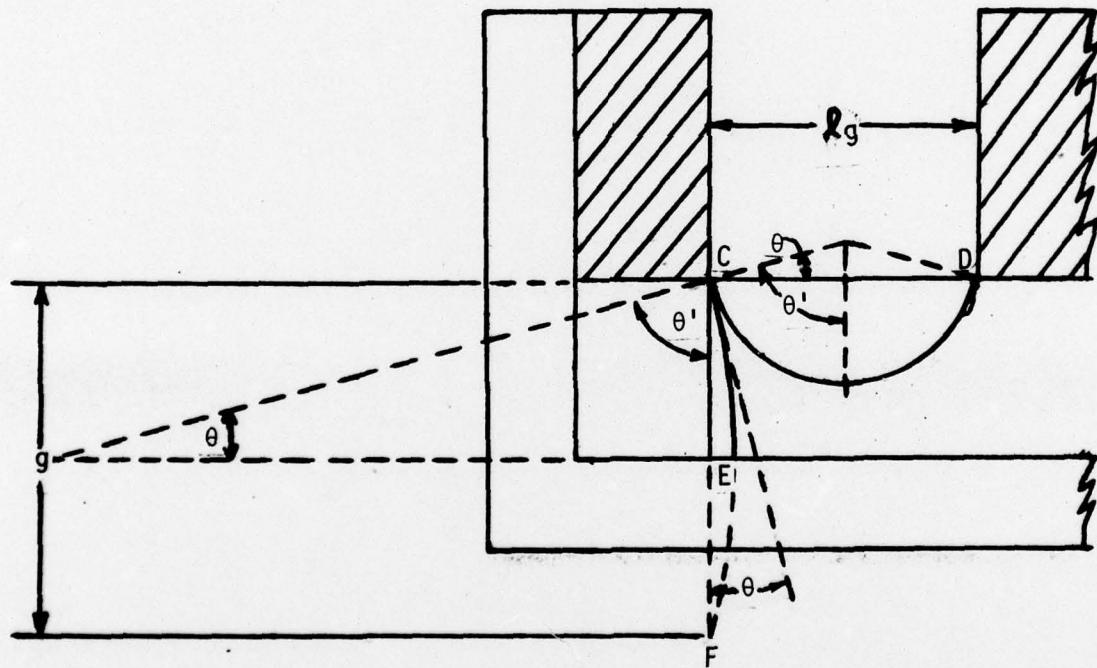


Fig. A-1. Determination of angle  $\theta$ .

From elementary geometry it is clear that

$$\frac{\theta'}{\sin \theta'} = \frac{\widehat{CD}}{\ell_g} = \frac{\widehat{2CE}}{\ell_g} = \frac{\widehat{CF}}{\ell_g} \quad (2A)$$

$$\frac{\theta}{\sin \theta} = \frac{\widehat{CF}}{\ell_g} \quad (3A)$$

Dividing (2A) by (3A) yields

$$\frac{\theta' \sin \theta}{\theta \sin \theta'} = \frac{g}{\ell_g} \quad (4A)$$

But  $\theta' = \frac{\pi}{2} - \theta$  and  $\frac{g}{\ell_g} = 1.333$ , so (4A) becomes

$$\frac{\left(\frac{\pi}{2} - \theta\right) \sin \theta}{\theta \cos \theta} = \left(\frac{\pi}{2\theta} - 1\right) \tan \theta = 1.333$$

$$\frac{\pi}{2\theta} - 1 = 1.333 \operatorname{ctn} \theta$$

$$\frac{\pi}{2\theta} = 1.333 \operatorname{ctn} \theta + 1 \quad (5A)$$

$$\frac{2\theta}{\pi} = \frac{1}{1.333 \operatorname{ctn} \theta + 1}$$

$$\theta = \frac{\pi}{2.67 \operatorname{ctn} \theta + 2} \quad (6A)$$

This is a transcendental equation which is solved graphically with the result that  $\theta = 16^\circ$ .

## APPENDIX B

### DETERMINATION OF THE FACTOR F IN THE CALCULATION OF $P_{MK}$

$P_{MK}$  is a standard gap permeance and would ordinarily be given by  $A/L$  where  $A$  and  $L$  are the gap-face area and gap length respectively. However, because  $P_{MK}$  emanates from the sides of the magnet, the driving MMF varies directly as the distance from the yoke end of the magnet such that MMF at the yoke is zero and achieves the full value of  $\frac{F_t}{2}$  at the other end. The factor of  $1/2$  appears because there are two magnets on the circuit and  $F_t$  is their total combined MMF. So the value of  $F$  at any point  $x$  from the yoke end of the magnet is given by

$$F = \frac{x}{L_M} \frac{F_t}{2} \quad (1B)$$

where  $L_M$  is the length of the magnet. The average value of  $x$  is at the middle of the path  $P_{MK}$  and can thus be written  $(\bar{x} = L_M - \frac{L_s}{2})$  where  $L_s$  is the path width and so

$$F = \frac{L_M - \frac{L_s}{2}}{L_M} \frac{F_t}{2} \quad (2B)$$

$$F = \frac{2L_M - L_s}{2L_M} \frac{F_t}{2} = P_{MK} F_t \quad (3B)$$

with

$$P_{MK} = \frac{2L_M - L_s}{4L_M} \quad (4B)$$